SETS, FUNCTIONS & RELATIONS

Definition: "Set" is synonymous with the words "collection", "aggregate", "class" and is comprised of elements. The words "element", "object" and "member" are synonymous.

If a is an element of a set A, then we write a $\in$ A. If is assumed here that if A is any set and a is any element, then either a $\in$ A or a $\notin$ A and the two possibilities are mutually exclusive. Thus one cannot say "consider the set A of some positive integers", because it is not sure whether 3 $\in$ A or 3 $\notin$ A.

Throughout this text we shall denote sets by capital Alphabets e.g. A, B, C, X, Y, Z etc. and the elements by small Alphabets e.g. a, b, c, x, y, z etc. The following are some examples of sets:

1. The collection of vowels in English Alphabets.
2. The collection of all past Presidents of the Indian Union.
3. The aggregate of all triangles in a plane.

The collection of good cricket players of India is NOT a set, since the term 'good players' is not well defined.

DESCRIPTION OF SETS

A set is often described in the following two ways. One can make use of any one of these two ways according to his (her) convenience.

1. **Roster Method (Tabular Form):** In this method a set is described by listing elements, separated by commas, within brackets, For example, the set of vowels of English Alphabet may be described as {a, e, i, o, u} The set of even natural can be described as {2, 4, 6,…} . Here the dots stand for 'and so on' Note that the order in which the elements are written makes no difference. Thus {a, e, i, o, u} denote the same set. Also, repetition of an element has effect, for example {1, 2, 3, 2} is the same set as {1, 2, 3}.

2. **Set Builder Method:** In this method, a set is described by a characterizing property, P(x) of its elements x. In such a case the set is described by {x P (x) holds} or {x : P(x) holds}, which is read as 'the set of all x such that P(x) holds'. The symbol '|' or ':' is read as 'such that'.

   In this representation the set of all even natural numbers can be written as : {x | x is natural number and x = 2n for n $\in$ z}.
   The set of all real numbers greater than – 1 and less than 1 can be described as : {x | x is a real number and $-1 < x < 1$}.
   A set is said to be empty or void or null set if it has no elements. Thus a set A is said to be empty set if the statement x $\in$ A is not true for any x. If A and B are any two empty sets, then x $\in$ A iff (if and only if) x $\in$ B is satisfied because there is no element x in either A or B to which the condition may be applied Thus A = B. hence there is only one empty set and we denote it by ϕ Therefore article 'the' is used before empty set.

FINITE AND INFINITE SETS

Finite set: A set A is called a finite set if it is either the void set or its elements can be listed (counted, labelled) by natural numbers 1, 2, 3,…..and the process of listing terminates at a certain natural number n (say).

The number n is called the **cardinality** (or order) of the set A is denoted by o(A) The void set is a finite set of cardinality zero. A set having only one element is called, a singleton set and its cardinality is one.

Infinite Set: A set whose elements cannot be listed by the natural numbers 1, 2, 3,.., n for any natural number n is called and infinite set.

EQUAL SETS

Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

If sets A and B are equal, we write A = B and $A \neq B$ when A and B are not equal.

Let A = {1, 2, 5, 6} and B = {5, 6, 2, 1,}. Then A = B because each element of A is an element of B and vice-versa.

SUBSETS

Let A and B be two sets. If every element of A is member of B, then A is called subset of B.

If A is subset of B, we write $A \subset B$, which is read as "A is a subset of B", "A is contained in B" Thus $A \subset B$ if $a \in A \Rightarrow a \in B$. (The symbol $\Rightarrow$ stands for "implies").

If A is a subset of B we also say that B contains A or B is a superset of A and we write $B \supset A$. If A is not a subset of B, we write $A \not\subset B$.

Obviously every set is a subset (superset) of itself and the void set ϕ is subset of every set. These two subsets are called improper subsets. A subset of a set A is called a proper subset of A if $S \neq A$ and we write $S \subset A$.

Thus, if S is a proper subset of A, then there exists an element $x \in A$ such that $x \notin S$.

It follows immediately from the definition that two sets A and B are equal iff $A \subset B$ and $B \subset A$. Thus whenever we want to prove that two sets A and B equal, we must prove that $A \subset B$ and $B \subset A$.

Theorem: Let A be a finite set having n elements. Then the total number of subsets of A is 2^{nd} number of proper subsets of A is $2^n - 1$.

FAMILY OR COLLECTION OF SETS

Let I be any set such that for each element $i \in I$ there is a set A_i. Then I is called and Index set and $\{A_i \mid i \in I\}$ is called family or collection of sets indexed by I.

POWER SET

Let A be a set. Then the family of all subsets of A is called the power set of A and is denoted by P(A). That is, $P(A) = \{S \mid S \subset A\}$.

Since the void set f and the set A itself are subsets of A and are therefore elements of P(A).

For example, let A = {1, 2, 3}. Then the subsets of A are f, {1}, {2}, {3}, {1, 2} {1, 3}, (2, 3) and {1, 2, 3} = A. Hence P(A) = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {A}.

If A is the void set ϕ, then P(A) has just one element, i.e., ϕ It immediately follows from the theorem that if A is a finite set having n elements, then P(A) has 2^n subsets.

UNIVERSAL SET

In any discussion in theory, there happens to be a set U that contains all sets under consideration Such as set is called the universal set. Thus a set that contains all sets in a given context is called the universal set. For example, in plane geometry the set of all points in the plane is the universal set.

Some Important Number Sets

N = Set of all natural numbers
= {1, 2, 3, 4,…}
Z or I Set of all integers
= {…–3, –2, –1, 0, 1, 2, 3,..}
Z^+ = Set of all + ve integers
= {1, 2, 3,…} = N
Z^- = Set of all – ve integers
= {– 1, – 2, – 3,..}
W = Set of all whole numbers
= {0, 1, 2, 3,…}
Z_0 = The set of all non-zero integers
= {± 1, ± 2, ± 3,..}
Q = The set of all rational numbers.

$$+ \quad \left\{\frac{p}{q} : p, q \text{ are integers, } q \neq 0 \text{ and } (p, q) = 1\right\}$$

R = The set of all real numbers.
R – Q = The set of all irrational numbers.
e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,… π ,e, log 2 etc. are all irrational numbers.

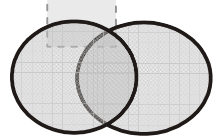
VENN DIAGRAMS

Sometimes pictures are very helpful in our thinking. In Venn-diagrams, the universal set U is represented by points with a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some elements, then to represent A and B we draw two intersecting circles.

OPERATIONS ON SETS

We shall now introduce some operations on sets to construct new sets from given ones.

Union: Let A and B be two sets. The union of A and B is the set of all those elements which belong either to A or to B or to both A and B. We shall use the notation $A \cup B$ (read as "A union B") to denote the union of A and B. Thus $A \cup B = \{x \mid x \in A \text{ or } x \in B)$ or $x \in A \cup B \Leftrightarrow x \in A \text{ or } x \in B$



If $\{A_i \mid i \in I\}$ be an arbitrary family of sets, then the set of all those elements which belong to A_i for some $i \in I$ is called the union of the family of sets and is denoted by $\bigcup_{i \in I} A_i$

Thus $\bigcup\limits_{i \in 1} A_i = \{x | x \in A_i$ for some $i \in I\}$. If $A_1, A_2, A_n\}$ is a finite family of sets, then their union is denoted by $\bigcup\limits_{i=1}^{n} A_1$ or $A_1 \cup A_2 \cup \ldots . \cup A_n$.

Intersection: Let A and B be two sets. The intersection of A and B is the set of all those elements that belong to both A and B.

The intersection of A and B is written as $A \cap B$ (read as "A intersection B") Thus $A \cap B = \{x | x \in A$ and $x \in B\}$ or $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$. If $A \cap B = f$, A and B are called disjoint sets. If $\{A_i | i \in I\}$is a family of sets, then the set of all elements x such that $x \in A_i$ for every $i \in I$ is called the interaction of the family of sets and is denoted by $\bigcap\limits_{i \in 1} A$

Thus $x \in \bigcap\limits_{i \in 1} A_i \Leftrightarrow x \in A$ for every $i \in I$.

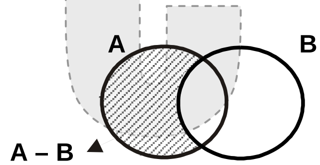
A family $\{A_i | i \in I\}$ is called a disjoint family of sets if any two sets of the family are disjoint

That is a $A_i \cap A_i = \phi$ for all $i, j \in I, i \neq j$.

If $\{A_1, A_2, \ldots . A_n\}$ is a finite family of sets, then their intersection is denoted by

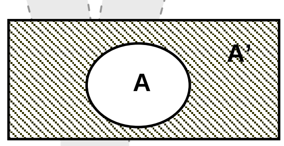
$$\bigcap_{i=1}^{n} A_i \text{ or } A_1 \cap A_2 \cap \ldots \cap A_n.$$

Difference: Let A and B be two sets. The difference of A and B, written as A – B, is the set of all those elements of A which do not belong to B. That is $A - B = \{x | x \in A$ and $x \notin B\}$ or $A - B = \{x \in A | x \notin B\}$. A.



Similarly the difference B – A is the set of all those elements of B that do not belong to That is $B - A = \{ x | x \in B$ and $x \notin A\}$.

Complement of a set: Let A and B be two sets such that $A \subset B$, then B – A is called the complement of A in B.



The complement of A in U, i.e. U – A is Simply called the complement A and is denoted by A' without any explicit reference of U. The shaded part represents A'.

Symmetric Difference of two sets: Let A and B be two sets. The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$. Thus $A \Delta B = (A - B) \cup (B - A)$.

SOME VERY IMPORTANT RESULTS

Theorem : Let A, B, C be any sets, then

(i) (a) $A \cup A = A$
(b) $A \cap A = A$ **Idempotent laws**

(ii) (a) $A \cup B = B \cup A$
(b) $A \cap B = B \cap A$ **Commutative laws**

(iii) (a) $A \cup (B \cup C) = (A \cup B) \cup C$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$ **Associative laws**

(iv) (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **Distributive laws**

Let A, B and X be any sets. Then

(i) $X - (X - A) = X \cap A$
(ii) $X - (A \cup B) = (X - A) \cap (X - B)$ **De Morgan's rules**
(iii) $X - (A \cap B) = (X - A) \cup (X - B)$

Corollary: Let A, B be any sets. Then
(i) $(A')' = A$
(ii) $(A \cup B)' = A' \cap B'$
(iii) $(A \cap B)' = A' \cup B'$

RELATIONS

Cartesian product of two sets: Let A, B be two sets. Then the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$, is called the Cartesian product of A and B, in that order, and is denoted by A x B. Thus A x B = {(a, b) | $a \in A$ and $b \in B$} For example, if A = {1, 2) and B = {a, b, c} then A x B = {(1, a), (1, b), (1, c), (2, a), (2, b) (2, c)} & B x A = {(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)}.

Since two ordered pairs (a, b) and (c, d) are equal iff a = c and b = d, therefore in general $A \times B \neq B \times A$.
If $A = \varphi$, then $A \times B = \varphi$.
If set A has m elements and B has n elements, then A x B has mn elements.
If $\{A_i \mid i \in n\}$ is a finite family of n sets, then their Cartesian product $A_1 \times A_2 \times \ldots \times A_n$ is the set of all n-tuples $(a_1, a_2, \ldots, a_n) \mid a_i \in A_i\}$. Obviously $(a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n) \Leftrightarrow a_i = b_i$ for all i.

The Cartesian product of the set $A = \{x \in R \mid 1 \leq x \leq 1\}$ with itself can be identified as a unit square plate. A right circular cylinder can be identified as the Cartesian product of the base circle and a generating line.
Binary Relation: Let A and B be two sets. Then a binary relation R from A to B is a subset of A x B. If A = B, then relation R from A to itself i.e. subset of A x A, then is called a relation on A (or in A).

If R is relation from a non-void set A to a non-void set B and if $(a, b) \in R$, then we write aRb (read a 'a is related to b by the relation R') If $(a, b) \notin R$, then we say that a is not related to b by a relation R. For example, R = {1, 2), (1, 3), (2, 3)} defines a relation on the set A = {1, 2, 3}.

Since $R \subset A \times A$. Here (1, 2), (1, 3) and (2, 3) $\in$ R, so we write 1 R 2, 1 R 3 and 2 R 3.

The total number of relations from a set A consisting of m elements to a set consisting n elements is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation A x B are trivial relations from A to B.
Domain and Range of a relation: Let A and B be two sets and let R be a relation from A to B. Then the sets $\{a \mid (a, b) \in R\}$ and $\{b \mid (a, b) \in R\}$ are called respectively the domain and range of the relation R.

In other words, the domain and the rang of a relation R form a set A to a set B are respectively the sets of all first and second components of ordered pairs in R.

For example, let R = {(a, 1), (a, 2), (A, 2), (b, 2) (c, 1)} be a relation from a set A = {a, b, c} to a set B = {1, 2, 3}. Then domain of R = {a, b, c} and range of R = {1, 2}.

Inverse of a relation: Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1} is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}.$$

Obviously $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$. Also domain of R = Range of R^{-1} and range of R = domain of R^{-1}.

For example, if R = {(a, 1), (a, 2), (c, 2)} is a relation from set A = {1, 2, 3) to a set B = {a, b, c}, then R^{-1} = {(1, a), (2, a), (2, c)} and domain R = {a, c} = Range R^{-1} and range R = {1, 2} = domain R^{-1}.

Identity relation: Let A be a set. Then the relation $I_A = \{(a, a) \mid a \in A\}$ on A is called the identity relation on A. Under the identity relation on A, each element of A is related to itself only.

For example, the relation {(1, 1), (2, 2), (3, 3)} is the identity relation on set A = {1, 2, 3).

Reflexive relation: A relation R on a set A is said to be reflective if $(a, a) \in R$ for all $a \in A$.

- The identity relation on a non-void set is reflexive.
- The universal relation on a non-void set is reflexive.
- The relation R_1 = {(1, 1) (1, 2), (2, 2) (3, 3) (3, 4) (4, 4) is a reflexive relation on set A = {1, 2, 3, 4}.
  But the relation R_2 = {(1, 1) (1, 2) (2, 2) (3, 4) (4, 4)} is not reflexive on A, since $3 \in A$, but $(3, 3) \notin R_2$.
- The relation on N defined by the phrase "x is greater than or equal to" is a reflexive relation. But the relation defined by the phrase" is greater than" is not reflexive.

Note that the identity relation on a set A is a ways reflexive. However, a reflexive relation need not the identity relation. Moreover, the identity relation on a set A is the smallest reflexive relation that can be defined on A and the universal relation is the largest reflexive relation on A.

Symmetric relation: Let A be a set. A relation R on A is said to be a symmetric relation.

$$\text{If } (a, b) \in R \Rightarrow (b, a) \in R \text{ for all } a, b, \in A.$$

- The identity and the universal relational on a non-void set are symmetric relations.
- Let L be the set of all lines in a plane and let r be a relation defined on L by the rule, $(x, y)\ R \Leftrightarrow x$ is perpendicular to y. Then R is a symmetric relation on L.

Transitive relation: Let A be any set relation R on A is said to a transitive relation if (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R for all a, b, c $\in$ A.

- The identity and the universal relations on a set A are transitive relations.
- In the set N of all natural numbers a relation R defined by phrase "x is greater than y" is a transitive relation.
- The relation "is perpendicular to" on the set L of all lines in a plane is not transitive.

Because if a line l_1 is perpendicular to a line l_2 and l_2 is perpendicular to l_3 then l_1 is parallel to l_3, i.e., l_1 is not perpendicular to l_3.

Anti-symmetric relation: The identity relation on a set is antisymmetric. (a, b) $\in$ R and (b, a) $\in$ R $\Rightarrow$ a = b for all a, b, c $\in$ A.

- The identity relation on a set is antisymmetric.
- The universal relation on a set A containing at least two elements is not antisymmetric, because if a $\neq$ b are in A, then a is related to b and b is related to a under universal relation but a $\neq$ b.
- The relation $\leq$ ("less than or equal to") on the set R or real numbers is antisymmetric, because a $\leq$ b and b $\leq$ a $\Rightarrow$ a = a = b for all a, b $\in$ R.

Equivalence relation: Let E be a relation on a set A. The E is said to be an equivalence on A if E is reflexive, symmetric and transitive.

Partial order relation: Let R be a relation on a set A. Then R is said to be a partial order relation on if is reflexive, antisymmetric and transitive.

Relation of Congruence modulo n: Let n be a fixed positive integer For (any a, b $\in$ Z, a is said to be congruent to b (modulo n) if n divides a – b. If a is congruent to b (modulo n), then we write a $\equiv$ b (mod n).

For example, 25 $\equiv$ 5 (mod 4) because 25 – 5 = 20 is divided by 4. But 25 is not congruent to 2 (mod 4) because 4 is not a divisor of 25 – 2 = 23.

Composition of Relations:

Let R and S be two relations from sets A to B and B to C respectively. Then we can define a relation SOR from A to C such that

(a, c) $\in$ SOR $\Leftrightarrow$ $\exists$ b $\in$ B s.t. (a, b) $\in$ r and (b, c) $\in$ S.

This relation is called the composition of R and S.

For example, if A = {1, 2, 3}, B = {a, b, c, d}, C = {p, q, r, s} be three sets such that R = {(1, a), (2, c), (1, c), (2, d)} is a relation from A to B and S = {(a, s), (b, q), (c, r)} is relation from B to C.

Then SOR is a relation from A to C given by

SOR = {(1, s) (2, r) (1, r)}

In this case ROS does not exist.

In general ROS $\neq$ SOR. Also $(SOR)^{-1} = R^{-1}OS^{-1}$.

<u>SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS</u>

If A, B and C are finite sets, and U be the finite universal set, then

(i) n (A $\cup$ B) = n(A) + n(B) – n(A $\cap$ B)

(ii) n (A $\cup$ B) = n(A) + n(B) $\Leftrightarrow$ A, B are disjoint non-void sets.

(iii) n (A – B) = n(A) – n (A $\cap$ B)
i.e. n(A – B) + n (A $\cap$ B) = n(A)

(iv) $n(A \Delta B)$ = Number of elements which belong to exactly one of A or B
$= n((A – B) \cup (B – A))$
$= n(A – B) + n(B – A)$ [$\because$ $(A – B)$ and $(B – A)$ are disjoint]
$= n(A) – n(A \cap B) + n(B) – n(A \cap B)$
$= n(A) + n(B) – 2\,n(A \cap B)$

(v) $(A \cup B \cup C) = n(A) + n(B) + n(C) – n(A \cap B) – n(B \cap C) – n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) – 3\,n(A \cap B \cap C)$.

(vii) Number of elements in exactly one of the sets A, B, C
$= n(A) + n(B) + n(C) – 2\,n(A \cap B) – 2\,n(B \cap C) – 2\,n(A \cap C) + 3\,n(A \cap B \cap C)$

(viii) $n(A' \cap B') = n((A \cap B)') = n(U) – n(A \cap B)$

(ix) $n(A' \cap B') = n((A \cap B)') = n(U) – n(A \cup B)$.

Ex. If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.

Sol. We have, $n(A \cup B) = n(A)$ and $n(B) – n(A \cap B)$.
This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is minimum respectively.
Case I: *When $n(A \cap B)$ is minimum, i.e. $n(A \cap B) = 0$.*
This is possible only when $A \cap B = \phi$. In this case,
$n(A \cup B) = n(A)$ *and* $n(B) – 0 = n(A) + n(B) = 3 + 6 = 9$.
So, maximum number of elements in $A \cup B$ is 9.
Case II: *When $n(A \cap B)$ is maximum.*
This is possible only when $\underline{A} \subseteq B$. In this case $n(A \cap B) = 3$.
$\therefore n(A \cup B) = n(A) + n(B) – n(A \cap B) = (3 + 6 – 3) = 6$.
So, minimum number of elements in $A \cup B$ is 6.

Ex. If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$

Sol. We have $A' \cap B' = (A \cup B)'$
$\therefore n(A' \cap B') = n((A \cup B)') = n(U) – n(A \cup B)$
$= n(U) – [n(A) + n(B) – n(A \cap B)]$
$= 700 – (200 + 300 – 100) = 300$.

Ex. In a town of 10,000 families it was found that 40% families buy newspaper A, 20 % families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three news papers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.

Sol. Let *P*, *Q* and *R* be the sets of families buying newspaper *A*, *B* and *C* respectively.
Let *U* be the universal set.
$n(P)$ = 40% of 10,000 = 4000,
$n(Q)$ = 20% of 10,000 = 2000,
$n(R)$= 10% of 10,000 = 1000,
$n(P \cap Q)$ = 5% of 10,000 = 500,
$n(Q \cap R)$ = 3% of 10,000 = 3000,
$n(R \cap P)$ = 4% of 10,000 = 400

$n(P \cap Q \cap R)$ = 2% of 10,000 = 200 and

$n(U)$ = 10,000

(i) Required number

$= (P \cap Q' \cap R') = n(P \cap (Q \cup R)')$

$= n(P) - n[P \cap (Q \cup R)]$ $\quad [\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(p) - n[(P \cap Q) \cup (P \cap R)]$

$= n(P) - [n(P \cap Q) + n(P \cap R) - n\{(P \cap Q) \cap (P \cap R)\}]$

$= n(P) - [n(P \cap Q) + n(P \cap R) - n(P \cap Q \cap R)]$

$= 400 - [500 + 400 - 200] = 3300$

(ii) Required number

$= n(P' \cap Q \cap R') = n(Q \cap P' \cap R')$

$= n(Q \cap (P \cup R)')$

$= n(Q) - n(Q \cap (P \cup R))$ $\quad [\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(Q) - n[(Q \cap P) \cup (Q \cap R)]$

$= n(Q) - [n(Q \cap P) + n(Q \cap R) - n\{(Q \cap P) \cap (Q \cap R)\}]$

$= n(Q) - [n(P \cap Q) + n(Q \cap R) - n(P \cap Q \cap R)]$

$= 2000 - [500 + 300 - 200] = 1400$

(iii) Required number

$= n(P' \cap Q' \cap R') = n[P \cup Q \cup R)']$

$= n(U) - n(P \cup Q \cup R)$

$= n(U) - [n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)]$

$= 10000 - [4000 + 2000 + 1000 - 500 - 300 - 400 + 200] = 4000.$

Examples:

1. Given A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, find $A \cup (B \cup C)$ and $(A \times B) \cap (B \times C)$

Sol. Clearly $A \cup (B \cup C)$ = {1, 2, 3, 4, 5, 6}. Now A x B = {(1, 3) (1, 4), (2, 3), (2, 4), (3, 3) (3, 4)} and B x C = {(3, 4), (3, 5), (3, 6) , (4, 4) (4, 5), (4, 6)}. Hence $(A \times B) \cap (B \times C)$ = {(3, 4)}.

2. If a N = {ax : $x \in N$). Describe the set $3N \cap 7N$.

Sol. According to the given notation, 3N = {3x : $x \in N$} = {3, 6, 9, 12, ….} and 7N = {7x : $x \cap N$} = {7, 14, 21, 28, 35, 42,…} Hence $3N \cap 7N$ = {21, 42, 63,…} = 21x : $x \in N$} = 21N.

3. (a) If A = {a, b, c, d}, B = {a, 2, 3}, find whether or not the following sets of ordered pairs are relations from A to B or not.

(i) R_1 = {(a, 1), (a, 3)}

(ii) R_2 = {(b, 1), (c, 2), (d, 1)}

(iii) R_3 = {a, 1), (b, 2), (3, 1)}

(b) If A = {1, 2, 3, 4}, define relations on A which have properties of being

(i) reflexive, transitive but not symmetric.

(ii) symmetric but neither reflexive nor transitive.

(iii) reflexive, symmetric and transitive

Sol. (a) (i) yes, (ii) yes, (iii) No. Ans.

(b) (i) We define a relation R_1 as R_1 = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 3), (1, 3)} Then it is easy to check that R_1 is reflexive, transitive but not symmetric.

(ii) Define R_2 as : $R_2 = \{(1, 2), (2, 1)\}$ It is clear that R_2 is symmetric but neither reflexive nor transitive. Write other relations of this type

(iii) We define R_3 as follows: $R_3 = \{(1, 1), (2, 2), (3, 3), (4, 4)\ (1, 2), (2, 1)\}$ Then evidently R_3 is reflexive, symmetric & transitive i. e R_3 is an equivalence relation on A.

4. Let $A = \{1, 2, 3\}$ and let $R_1 = \{(1,1), (1, 3), (3, 1)\ , (3, 1)\ (2, 2), (2, 1), (3, 3)\}$

$R_2 = \{(2, 2), (3, 1), (1, 3)\}$

$R_3 = \{(1, 3), (3, 3)\}$ $R_4 = A \times A$

Find whether or not each of the relations R_1, R_2, R_3, R_4, on A is

(i) reflexive (ii) symmetric (iii) transitive

Sol. R_1 is reflexive but neither symmetric nor transitive. R_2 is symmetric but neither reflexive nor transitive. R_3 is transitive but neither symmetric nor reflexive. R_4 is reflexive, symmetric and transitive that is R_4 is an equivalence relation

5. If R be a relation from $A = \{1, 2, 3, 4\}$ to $B = \{1, 3, 5\}$, such that a R b i.e. $(a, b) \in R$ iff $a < b$, then ROR^{-1} is

(1) $\{(1, 3), (1, 5), (2, 3)\ (2, 5), (3, 5), (4, 5)\}$

(2) $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

(3) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$

(4) $\{(3, 3), (3, 4), (4, 5)\}$

Sol. Ans. (3) we have

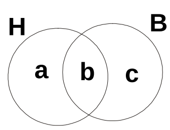
$R = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)$

$\therefore R^{-1} = \{3, 1), (5, 1), (3, 2), (5, 2), (5, 3),(5, 4)\}$

Hence $ROR^{-1} = \{(3, 3), (3, 5), (5, 3), (5, 5)\}$

6. In a group of 1000 persons, 760 can speak Hindi & 430 can speak Bengali.

(a) How many can speak both? (b) How many can speak Hindi only & Bengali only?



Sol. Let H denote Hindi, B denote Bengali

We have $a + b = 760$, $b + c = 430$, $a + b + c = 1000$

$\Rightarrow a = 570$, $c = 240$ or $b = 190$ or 190 people can speak both 570 only Hindi & 240 only Bengali.

<u>Alternative Method:</u>

$n(H \cup B) = n(H) + n(B) - (H \cap B) \Rightarrow 1000 = 760 + 430 - n(H \cap B) \Rightarrow n(H \cap B) = 190$ etc.

7. In a certain city only two newspapers A & B are published It is known that 25% of the city population reads A & 20% read B while 8% read both A & B It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements while 50% of those who read both A & B look into advertisements. What percentage of the population read an advertisement?

Sol. Let A & B denote sets of people who read paper A & paper B resp,

then $n(A) = 25$, $n(B) = 20$, $n(A \cap B) = 8$. Hence $n(A - B) = n(A) - n(A \cap B) = 25 - 8 = 17$

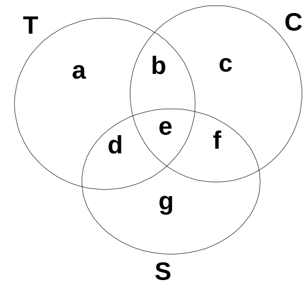
$n(B - A) = n(B) - n(A \cap B) = 20 - 8 = 12$

Now percentage of people reading an advertisement
= [(30% of 17) + (40% of + 12) + (50% of 8)]% = 13.9 % Ans.

8. In a survey of 2000 students, 48% used coffee (C), 54% used tea (T) and 64% smoked (S). Of the total, 28% used C & T, 32% used T & S and 30% used C & S. 6% did not like any of the three. Find:

(i) The number using all three (ii) T & S but not C
(iii) T but not C (iv) only C

Sol. We have b + c + e + f = 960 (48%)
a + b + c + d = 1080
g + d + e + f = 1280
b + e = 560, d + e = 640, e + f = 600
a + b + c + d + e + f + g = 1880 (94%)
Solve to find
(i) c = 360
(ii) d = 280
(iii) a + d = 520
(iv) g = 160 Ans.



<u>Alternative Method:</u>

n (C ∪ T ∪ S) = n(C) + n(T) + n(S) – (C ∩ T) – (C ∩ S) – n (T ∩ S) + n(C ∩ T ∩ S)
⇒ 1880 = 960 + 1080 + 1280 – 560 – 640 – 600 + n(C ∩ T ∩ F)
⇒ n(C ∩ T ∩ S) = 360 (This value is basically the "c" of the first method).
Now from the figure, all the values can be determined one by one.